

EFFECT OF VARIATIONS IN IRU INTEGRATION TIME INTERVAL ON ACCURACY OF AQUA ATTITUDE ESTIMATION*

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ABSTRACT

The Earth Observing System (EOS)-PM1 Aqua satellite is the second of the series of EOS satellites, which are part of NASA's Earth Science Enterprise Program.

During Aqua launch support, attitude analysts noticed several anomalies in Onboard Computer (OBC) rates and in rates computed by the ground Attitude Determination System (ADS). These included: i) periodic jumps in the OBC pitch rate every 2 minutes; ii) spikes in ADS pitch rate every 4 minutes; iii) close agreement between pitch rates computed by ADS and those derived from telemetered OBC quaternions (in contrast to the step-wise pattern observed for telemetered OBC rates); iv) spikes of ± 10 milliseconds in telemetered IRU integration time every 4 minutes (despite the fact that telemetered time tags of any two sequential IRU measurements were always 1 second apart from each other). An analysis presented in the paper explains this anomalous behavior by a small average offset of about 0.50 ± 0.05 μ sec in the time interval between two sequential accumulated angle measurements. It is shown that errors in the estimated pitch angle due to neglecting the aforementioned variations in the integration time interval by the OBC is within ± 2 arcseconds. Ground attitude solutions are found to be accurate enough to see the effect of the variations on the accuracy of the estimated pitch angle.

INTRODUCTION

The Earth Observing System (EOS)-PM1 Aqua satellite is the second of the series of EOS satellites, which are part of NASA's Earth Science Enterprise Program. Aqua was launched on May 4, 2002 into a nearly-circular, frozen, sun-synchronous, polar orbit of about 98.2 degrees inclination and an altitude of about 705 km. Under nominal conditions its attitude control is performed by an onboard Kalman filter using measurement data from 2 Star Tracker Assemblies (STA) and an Inertial Reference Unit (IRU). A more detailed description of the mission can be found in References 1 and 2.

During Aqua launch support, attitude analysts noticed several anomalies in telemetered Onboard Computer (OBC) rates and in IRU rates computed by the ground Attitude Determination System (ADS):

- i) periodic jumps in the OBC pitch rate every 2 minutes (Figure 1a);
- ii) spikes in ADS pitch rate every 4 minutes (Figure 1b);

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- iii) close agreement between pitch rates computed by ADS and those derived from telemetered OBC quaternions (Figures 1c), in contrast to the step-wise pattern observed for telemetered OBC rates in Figure 1a;
- iv) spikes of ± 10 milliseconds in telemetered IRU integration time every 4 minutes (Figure 1d), despite the fact that (with an accuracy of a few microseconds) telemetered time tags of any two sequential IRU measurements were always 1 second apart from each other.

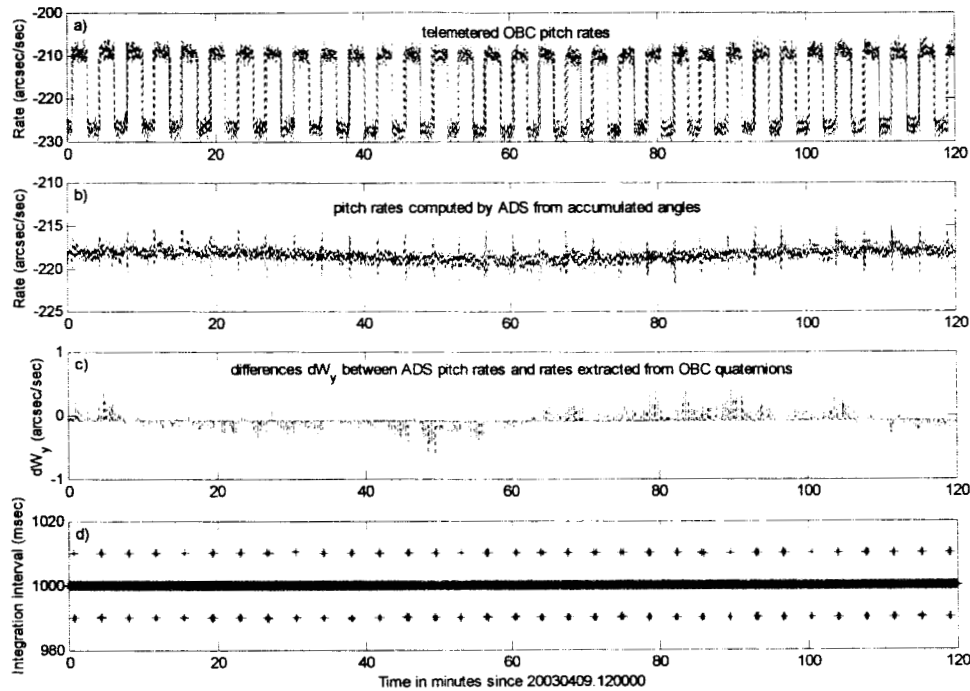


Figure 1. Anomalous behavior of adjusted IRU pitch rates and associated integration time interval

An analysis presented in the paper explains this anomalous behavior by a small offset of about $0.5 \mu\text{sec}$ in the time interval between two sequential accumulated angle measurements. It is shown that errors in the estimated pitch angle due to neglecting the aforementioned variations in the integration time interval by OBC are within ± 2 arcseconds. Ground attitude solutions are found to be accurate enough to see the effect of the variations on the accuracy of the estimated pitch angle.

SOME DRAWBACKS IN ONBOARD PROCESSING OF IRU TIME TAGS

A detailed analysis of the onboard algorithm³ revealed that all the mentioned anomalies were caused by the known drawback⁴ in onboard processing of accumulated angle measurements and their time tags. The time interval between accumulated angle measurements used to compute onboard IRU rates was set *by design*³ to the OBC minor cycle duration of 125 milliseconds, instead of using a precise integral multiple of the gyro measurement cycle (nominally 10 milliseconds per gyro sample⁴). There are either 12 or 13 gyro measurements per minor cycle resulting in either 120 or 130 milliseconds integration time.⁴ Use of the minor cycle time of 125 milliseconds, instead of the actual integration time, causes relative errors of 4% in OBC body rates, which corresponds to an absolute error of about 9 arcsec/sec in the nominal pitch rate. The

resultant absolute errors for the nominal roll and yaw body rates are at least an order of magnitude smaller, whereas the mission requirements for the attitude accuracy during maneuvers are much less restrictive. The whole picture is then additionally complicated by the fact that the actual time between two sequential accumulated angle measurements slightly differs from the nominal value⁴ of 10 milliseconds.

Let us first show that the actual integration time would always change by ± 10 milliseconds from one minor cycle to another, if the time interval between any two sequential accumulated angle measurements were precisely 10 milliseconds. Figure 2 illustrates the pattern for three sequential minor cycles. It is assumed that the OBC collects 13 measurements within the k^{th} minor cycle, which are marked in Figure 2 by circles. Let t_k and t_{kj} be, respectively, the start time of the k^{th} minor cycle and the time of the j^{th} accumulated angle measurement within this minor cycle. (Note that t_k is also the end time of the $(k-1)^{\text{th}}$ minor cycle.) Since $t_{k,13} - t_{k,1} = 120$ msec, the time of the first measurement from the start of the k^{th} minor cycle ($\Delta t_{k,1} = t_{k,1} - t_k$) may not exceed 5 milliseconds. Unless $\Delta t_{k,1}$ is precisely equal to 5 milliseconds, the time interval between the last measurement within the $(k-1)^{\text{th}}$ minor cycle and the end of this minor cycle must be larger than 5 milliseconds, and therefore, the OBC may collect only 12 measurements within the minor cycle in question. By analogy, keeping in mind that $t_{k+1} - t_{k,13} < 5$ msec, one can verify this is also true for the $(k+1)^{\text{th}}$ minor cycle.

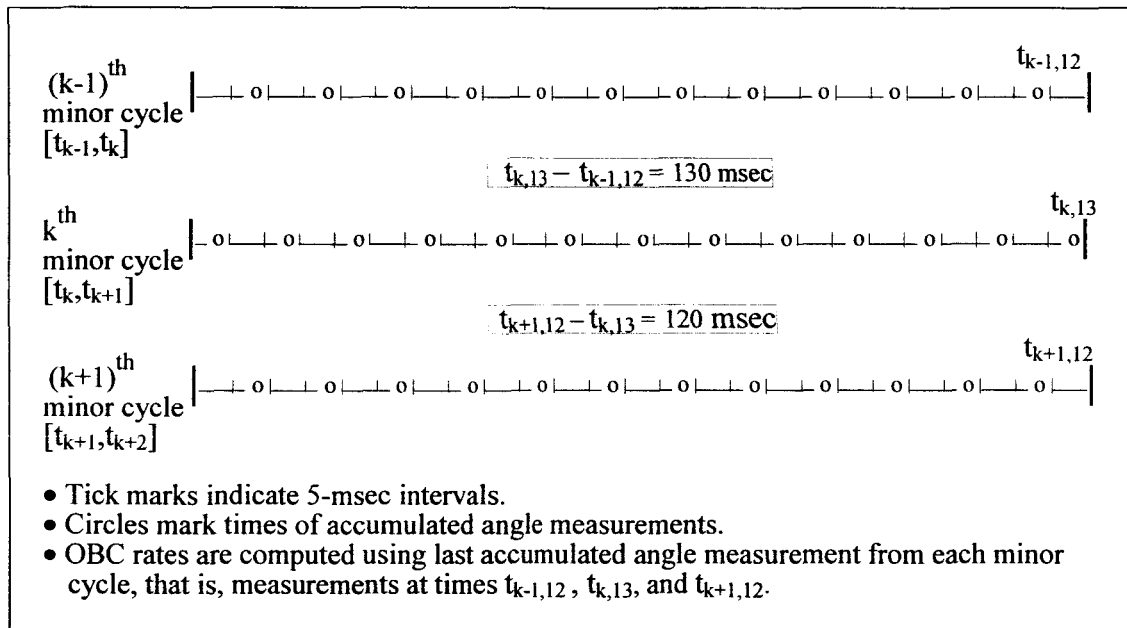


Figure 2. Periodic variations of actual integration time for three consecutive minor cycles assuming that the gyro measurement cycle is precisely 10 msec

By using the integration time of 125 milliseconds, instead of its actual value (120 or 130 milliseconds), the OBC either overestimates or underestimates, respectively, the magnitude of the pitch rate by 4%. If the time interval between sequential accumulated angle measurements were precisely equal to 10 milliseconds, then this alternation of error sign would last forever. Since OBC rates are telemetered with a frequency of 1 Hertz and are always taken from the same minor cycle of the 1-second cycle, this would imply that telemetry must contain either only underestimated or overestimated values of the pitch rate. On the contrary, as shown in Figure 1a, telemetered OBC pitch rates jump either up or down each 2 minutes. These jumps cannot be

explained unless the time interval between two sequential measurements slightly deviates from 10 milliseconds.

As mentioned above, each of the 4-minutes jumps in the telemetered pitch rate are accompanied by spikes in the integration time. Similar spikes appear in pitch rates computed from telemetered accumulated angles, using the 125-msec integration time interval. Note that the jumps in telemetered pitch rates in Figure 3a perfectly correlate with the spikes in Figures 3b and 3c as well as with jumps in the OBC pitch error computed from telemetered OBC quaternions and depicted in Figure 3d. (An algorithm used to compute this error will be discussed in the next Section.)

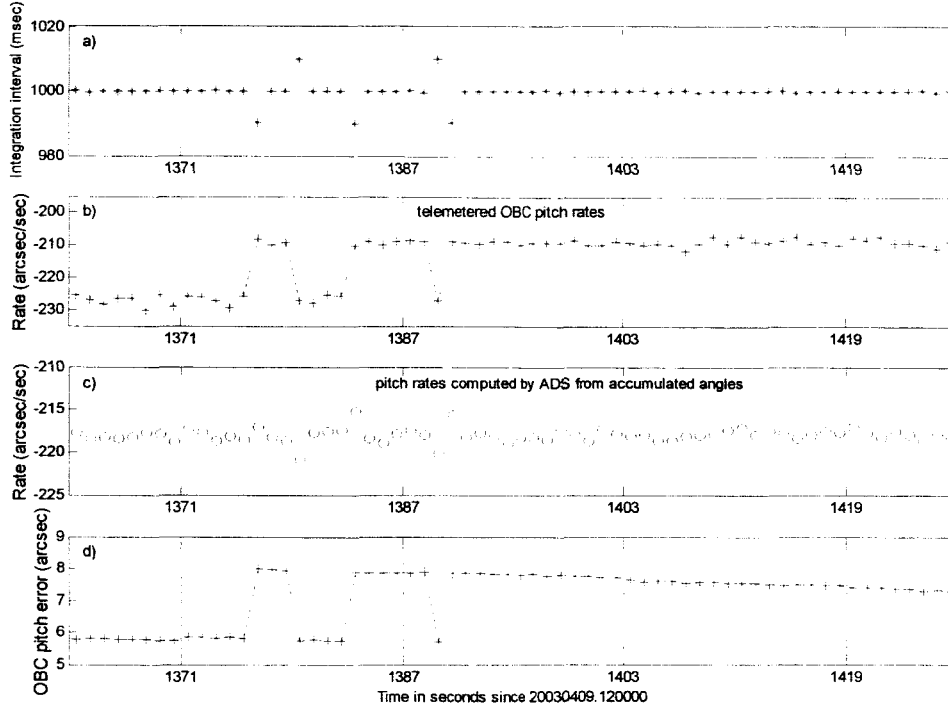


Figure 3. Correlation between irregularities in data extracted from different mnemonics

To qualitatively explain these anomalous jumps and spikes, let us assume that the time interval between two sequential accumulated angle measurements has a constant time offset δ , so that the gyro measurement cycle is equal to $10 \text{ msec} + \delta$. If this time offset is positive, then there may be two sequential minor cycles (k and $k+1$, for example) such that only 12 measurements are collected within each of these two 125-msec intervals. This happens when the first measurement in the minor cycle falls within the range:

$$10 \text{ msec} - 24\delta < \Delta t_{k,1} < 10 \text{ msec} + \delta. \quad (1)$$

It may be shown by keeping in mind that times of measurements from the start of the k^{th} minor cycle are related to each other via the relation:

$$\Delta t_{k,j} \equiv t_{k,j} - t_k = \Delta t_{k,1} + (j-1) \times (10 \text{ msec} + \delta), \quad (2)$$

so that

$$t_{k+1} - t_{k,12} = 125 \text{ msec} - \Delta t_{k,12} < 5 \text{ msec} - 13\delta \quad (3)$$

(taking into account that t_{k+1} is both the end of the k^{th} minor cycle and the start of the $(k+1)^{\text{th}}$ minor cycle). This implies that the time of the first measurement from the start of the $(k+1)^{\text{th}}$ minor cycle, $\Delta t_{k+1,1}$, must satisfy the inequality:

$$\Delta t_{k+1,1} > 5 \text{ msec} - 12\delta. \quad (4)$$

One can thus verify that

$$\Delta t_{k+1,12} > 115 \text{ msec} - \delta, \quad (5)$$

and hence

$$t_{k+2} - t_{k+1,12} = 125 \text{ msec} - \Delta t_{k+1,12} < 10 \text{ msec} + \delta. \quad (6)$$

The inequalities (3) and (6) explicitly show that there are exactly 12 measurements within each of the minor cycles.

Similar arguments are applied to a negative time offset, $\delta < 0$. In this case there may be two sequential minor cycles (k' and $k'+1$, for example) such that only 13 measurements are collected within each of these two 125-msec intervals. This happens when the first measurement in the minor cycle falls within the range

$$0 < \Delta t_{k',1} < 25 |\delta|. \quad (7)$$

and therefore,

$$\Delta t_{k'+1,1} < 5 \text{ msec} - 12|\delta|, \quad (8)$$

so that

$$\Delta t_{k'+1,13} < 125 \text{ msec}. \quad (9)$$

To understand why some jumps in OBC pitch rates are not accompanied by spikes in the telemetered integration time, remember that this integration time is computed for 1-second time intervals, by summing up time steps between accumulated angle measurements over 4 pairs of odd and even minor cycles. These 250-msec ‘double cycles’ usually contain 25 measurements so that the integration time ΔT is precisely equal to 1000 milliseconds. As seen from Figure 4, jumps in telemetered values of the OBC pitch rate were not accompanied by spikes in the telemetered integration time if two sequential minor cycles with the same number of measurements belong to different double cycles. As explained above, the magnitude of the IRU rates is overestimated onboard by the factor 1.04 for minor cycles with 13 measurements and underestimated by the factor 0.96 otherwise. This implies that telemetered values of the OBC pitch rate are equal to either -1.04 RPO or to -0.96 RPO, depending on the number of measurements in the minor cycle used to compute IRU rates for the given 1-second interval. For

purely illustrative purposes (see comments below), Figure 4 depicts values of OBC rates for the 4th minor cycle in each 1-second interval.

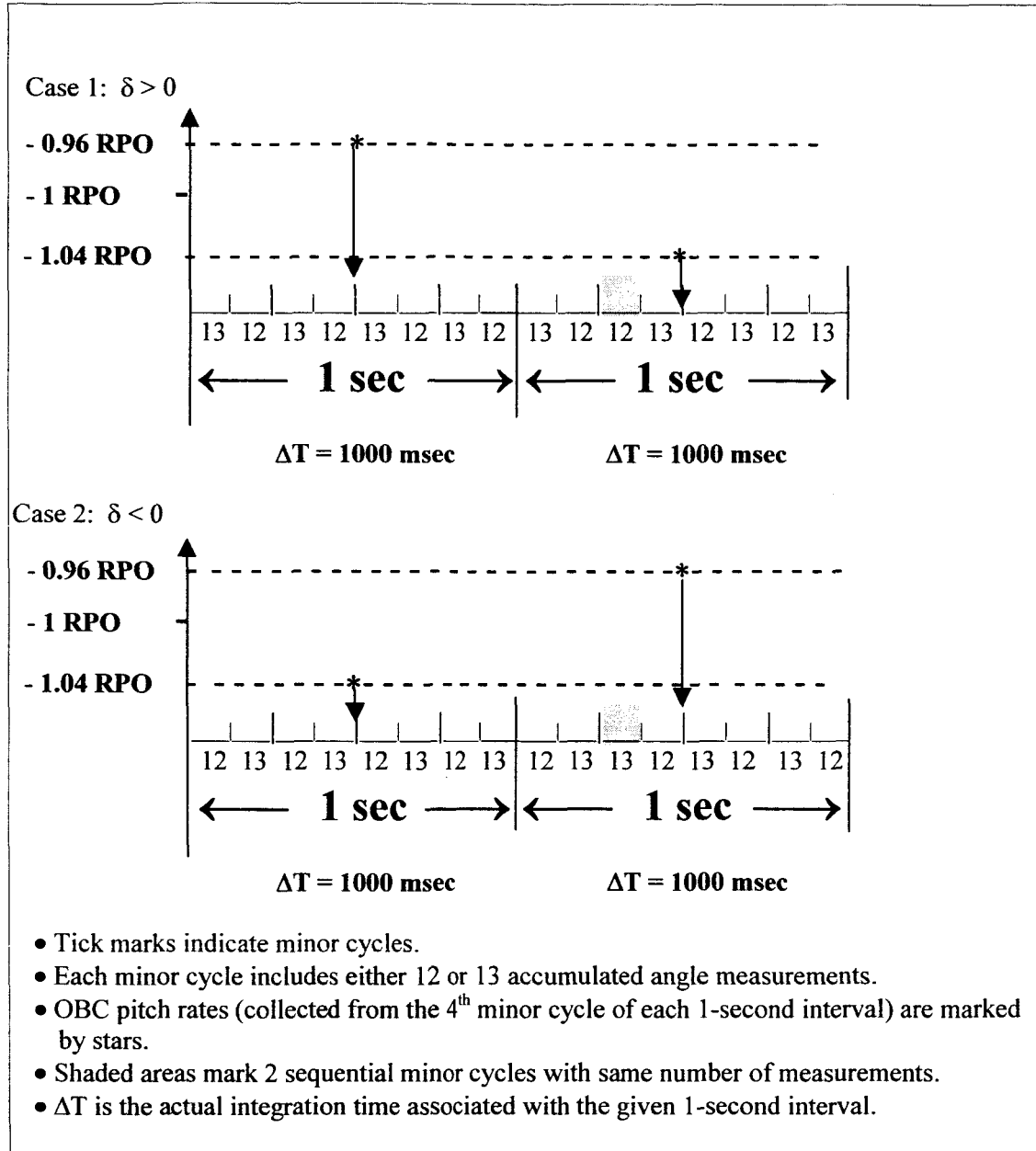


Figure 4. Rate jumps not accompanied by spikes in the telemetered integration time

Making use of inequalities (1), (4), (7), and (8), one can easily verify that the first measurements in the darkened minor cycles in Figure 4 (the second minor cycle in each shaded area) lie within the $25|\delta|$ intervals:

$$5 \text{ msec} - 12\delta < \Delta t_{k+1,1} < 5 \text{ msec} + 13\delta \quad \text{for } \delta > 0 \quad (10a)$$

and

$$5 \text{ msec} - 37|\delta| < \Delta t_{k'+1,1} < 5 \text{ msec} - 12|\delta| \quad \text{for } \delta < 0. \quad (10b)$$

We thus conclude that both upward and downward jumps in telemetered values of the OBC pitch rate without a spike in the telemetered integration time occur when the first measurement is near 5 milliseconds from the start of a double cycle. Since the OBC pitch rate usually jumps up and down a few times before stabilizing near -1.04 RPO (see the region near 25 minutes in Figure 5a), the time offset δ must change its sign. According to Figure 4, it should be biased toward positive values because the number of downward jumps (rate decrease) is always larger than the number of upward jumps (rate increase) for total jumps from -0.96 RPO down to -1.04 RPO.

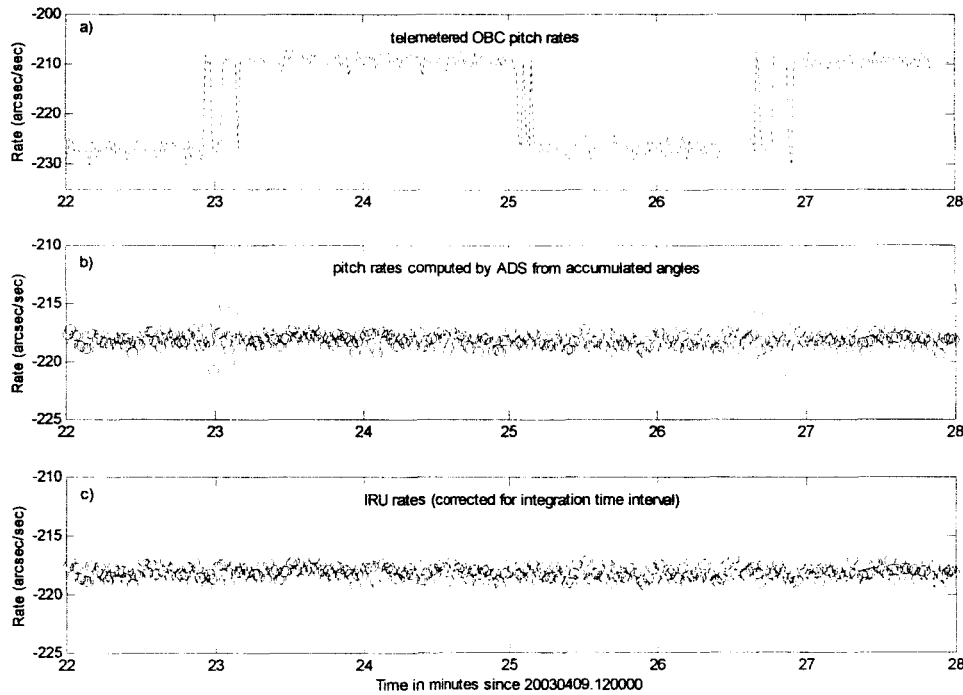


Figure 5. Comparison between telemetered pitch rates and ground IRU pitch rates before and after adjustment for variations in the integration time interval

Jumps in telemetered values of the OBC pitch rate are accompanied by spikes in the telemetered integration time when minor cycles with the same number of measurements belong to the same double cycle. Figure 6 illuminates this scenario for both positive and negative time offsets. An analysis of Figures 3a and 3b shows that 990-msec and 1010-msec integration times are associated with upward and downward jumps in the telemetered OBC pitch rates. For this reason we concluded telemetered rates must be collected from one of the even minor cycles in 1-second time intervals. In Figure 6 we chose it to be the 4th minor cycle for purely illustrative purposes.

It is essential that the number of negative spikes in the integration time is always larger than the number of positive spikes. For example, there are 3 downward and only 2 upward spikes in Figure 3a. This observation perfectly agrees with our previous conclusion that the time offset δ is biased toward positive values.

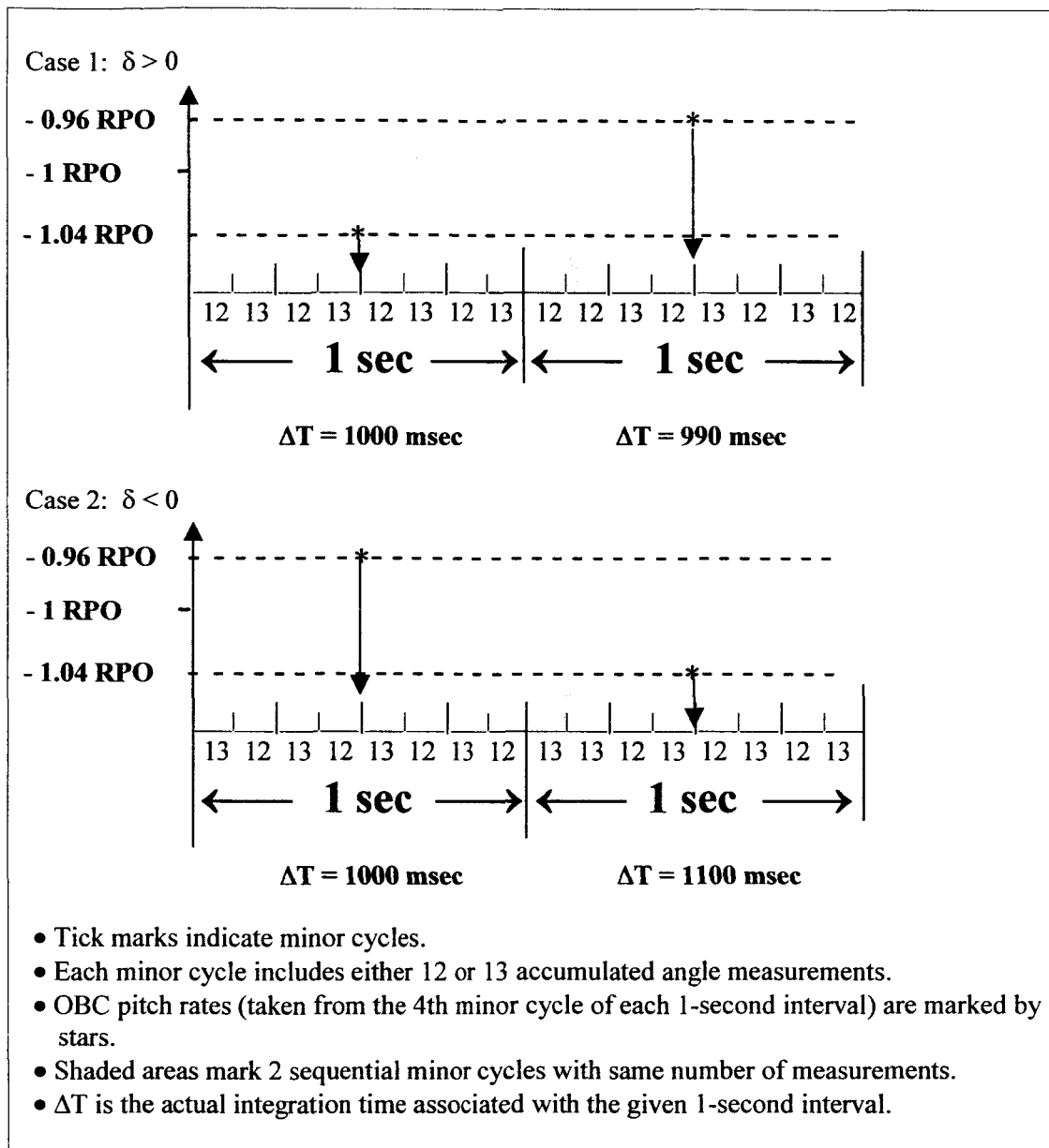


Figure 6. Rate jumps accompanied by spikes in the telemetered integration time

Since the OBC computes IRU rates each minor cycle, it first seems puzzling that spikes in pitch errors extracted from OBC quaternions correlate with spikes in the telemetered integration time, ΔT (compare Figures 3a and 3c). However, it should be noticed that OBC quaternions are propagated over a 1-second interval in eight 125-msec steps. Usually the OBC pitch rate is overestimated in one of any two sequential minor cycles and underestimated in another so that the appropriate errors compensate each other. This compensation does not occur if there are double cycles with either 24 or 26 measurements, which results in a spike in the OBC pitch error. This propagation error remains uncorrected until the next Kalman filter update, which occurs every 16 seconds

The next question to address is why the upward and downward jumps in the telemetered pitch rate alternate with a nearly constant frequency and how this frequency is related to an average value of time offsets $\bar{\delta}$. To answer this question, let us return to the inequality (1), which determines the range for the first accumulated angle measurement in a pair of two sequential minor cycles with the same number of measurements. Note that the magnitude of telemetered body rates remains either overestimated or underestimated until the time of the first measurement again reaches the range $(10 \text{ msec} - 24 \bar{\delta}, 10 \text{ msec} + \bar{\delta})$ from the start of a $(k+2n+1)^{\text{th}}$ minor cycle. Since the time of the first measurement from the start of the minor cycle increases by $25\bar{\delta}$ after each two minor cycles, the use of the inequality (4) gives

$$10 \text{ msec} - 24 \bar{\delta} < 5 \text{ msec} + (25n - 12) \bar{\delta} < 10 \text{ msec} + \bar{\delta} \quad (11)$$

so that

$$0.2 \text{ msec} - 0.48 \bar{\delta} < n \bar{\delta} < 0.2 \text{ msec} + 0.52 \bar{\delta}. \quad (12)$$

Since $\bar{\delta}$ is expected to be very small, it can be thus approximated as

$$\bar{\delta} \sim 0.2 \text{ msec} / n. \quad (13)$$

Using $n = 4 \times 120 = 480$, every-2-minute-upward or-downward jump in telemetered values of the pitch rate can be qualitatively explained by an average time offset $\bar{\delta}$ of about $0.42 \mu\text{sec}$. (This is just a preliminary value; a more accurate statistical estimate is discussed below.)

The only issue left to explain is why spikes in the telemetered integration time appear only every 4 minutes. Let us count minor cycles starting from a 1-second interval with the 990-msec integration time. The double cycle with 24 measurements causing the spike is represented in Table 1 by columns $k = k_0$ and $k = k_0 + 1$ (see inequalities (1) and (4) above). If $k_0 = 1$ or 3, the OBC rate collected from the 4^{th} minor cycle of each 1-second interval also jumps during the first second; if $k_0 = 5$ or 7, it jumps 1 second later.

With each double cycle, the time of the first measurement from the start of the even minor cycle moves forward by $25 \bar{\delta}$ until it reaches the interval $(10 - 24 \bar{\delta}, 10 + \bar{\delta})$ at $N_{\uparrow\downarrow}$ seconds. $N_{\uparrow\downarrow}$ represents the time in seconds between two sequential jumps in the telemetered pitch rate, with the two arrows indicating the direction and sequence of the rate change. To force the $(8N_{\uparrow\downarrow} + k_1 + 2)^{\text{th}}$ minor cycle to be within the anomalous 1-second cycle, we choose k_1 to be negative ($k_1 = -3$ or -1), if $k_0 = 5$ or 7, and positive ($k_1 = 1$ or 3), if $k_0 = 1$ or 3. The second spike in the telemetered integration time appears $N_{\vee\vee}$ seconds later after the first spike. Note that $N_{\vee\vee} \approx N_{\uparrow\downarrow} + N_{\downarrow\uparrow}$.

One thus finds

$$100 N_{\uparrow\downarrow} \bar{\delta} \approx 5, \quad (14)$$

$$100 N_{\vee\vee} \bar{\delta} \approx 10. \quad (15)$$

Table 1. Lower bounds for the first accumulated angle measurements from the start of minor cycles.

A spike in the integration time at 1 second (accompanied by an upward jump in telemetered pitch rate)				
k (minor cycle #)	$k_0 = 1, 3, 5, \text{ or } 7$	$k_0 + 1$	$k_0 + 2$	$k_0 + 3$
Lower bound (msec)	$10 - 24 \bar{\delta}$	$5 - 12 \bar{\delta}$	0	$5 + 13 \bar{\delta}$
Number of measurements	12	12	13	12
A downward jump in telemetered pitch rate at $N_{\uparrow\downarrow} + 1$ seconds				
k (minor cycle #)	$8N_{\uparrow\downarrow} + k_1$ ($k_1 = -1, 1, 3, \text{ or } 5$)	$8N_{\uparrow\downarrow} + k_1 + 1$	$8N_{\uparrow\downarrow} + k_1 + 2$	$8N_{\uparrow\downarrow} + k_1 + 3$
Lower bound (msec)	$5 - 37 \bar{\delta}$	$10 - 24 \bar{\delta}$	$5 - 12 \bar{\delta}$	0
Number of measurements	13	12	12	13
The second spike in integration time at $N_{\vee} + 1$ seconds (accompanied by the second upward jump in telemetered pitch rate)				
k (minor cycle #)	$8N_{\vee} + k_2$ ($k_2 = 1, 3, 5 \text{ or } 7$)	$8N_{\vee} + k_2 + 1$	$8N_{\vee} + k_2 + 2$	$8N_{\vee} + k_2 + 3$
Lower bound (msec)	$10 - 24 \bar{\delta}$	$5 - 12 \bar{\delta}$	0	$5 + 13 \bar{\delta}$
Number of measurements	12	12	13	12

The average time without spikes in the integration time interval is found to be equal to $N_{\vee} = 209$ seconds (with standard deviation of 5 seconds), which is slightly less than 4 minutes mentioned at the beginning of the paper. An average time without jumps in the telemetered pitch rate also varies depending on which upward or downward jump comes first, namely, their average durations in seconds are equal to $N_{\uparrow\downarrow} = 112$ and $N_{\downarrow\uparrow} = 91$, respectively. Note also that N_{\vee} is slightly larger than $N_{\uparrow\downarrow} + N_{\downarrow\uparrow}$, since the time interval without spikes also includes the region between the first and last downward jumps in the middle of the interval.) Substituting the cited values of $N_{\uparrow\downarrow}$ and N_{\vee} in Eqs. (14) and (15), one finds two slightly different values for $\bar{\delta}$: $\sim 0.45 \mu\text{sec}$ and $\sim 0.49 \mu\text{sec}$, respectively, whereas using a similar formula

$$100 N_{\downarrow\uparrow} \bar{\delta} \approx 5, \quad (16)$$

for $N_{\downarrow\uparrow}$ gives a value of $0.55 \mu\text{sec}$. Combining these three values gives $\bar{\delta} \approx 0.50 \pm 0.05 \mu\text{sec}$.

We see that introducing a slightly varying time offset for the time interval between two sequential accumulated angle measurements explains most of anomalies illustrated by Figures 3a, 3b, 3c, 5a, and 5b. There are however a few puzzles, which the author was unable to interpret, namely,

- i) an average spread of multiple jumps in the telemetered pitch rate is larger when upward jumps dominate (13.5 sec and 5.7 sec, respectively);
- ii) duration of the time interval between 2 sequential jumps in the telemetered pitch rate is noticeably larger when the a rate increase occurs first (~112 seconds and ~91 seconds, respectively);
- iii) the total number of multiple jumps is always odd.

Concerning item iii): it seems most probable that the total number of jumps at the beginning of each interval will be odd, but there is no apparent reason why this rule cannot be occasionally violated.

USE OF IRU RATES FOR VARIATIONS IN INTEGRATION TIME INTERVALS

The correction of IRU rates $\bar{\omega}_i$ for time intervals of the integration that deviate by ΔT from 1 second is performed using the formula:

$$\bar{\omega}_i^{\text{corr}} = \bar{\omega}_i / \Delta T, \quad (17)$$

Figs. 5b and 5c compare IRU pitch rates before and after the correction. As expected, all spikes disappear after variations in the integration time interval are explicitly taken into account.

The corrected IRU rates, $\bar{\omega}_i^{\text{corr}}$, were used to re-calculate a ground attitude history using the batch least-squares estimator. This attitude history is displayed in Fig. 3c to show the telemetered pitch errors that result from neglecting variations in the integration time interval onboard. This plot depicts differences between OBC pitch angles and corresponding values estimated using corrected IRU rates. The slowly changing bias between the two solutions has several causes, including an onboard error in time tags of star tracker measurements, the inability of the batch least-squares estimator to account for time-varying gyro biases, and temperature-dependent variations of the relative star-tracker misalignment. The 2-arcsecond jump in the OBC pitch angle is caused solely by the onboard use of the 125-msec time interval, instead of its actual value of 120 or 130 milliseconds.

Usually alternating positive and negative errors in onboard IRU rates compensate each other during the attitude propagation, but this compensation fails when the alternating pattern breaks leading to noticeable attitude errors. As seen from Fig. 7a, the error slowly decreases with time as the onboard Kalman filter is updated with star tracker measurements but it takes up to 2 minutes for the OBC to completely correct this error.

It is interesting that the Kalman filter updates with star tracker measurements become pronounced only after the error is corrected. In fact, steps at 16-second intervals (caused by the updates) become evident approximately 2 minutes after the first jump. (The updates are marked by vertical lines in Figure 3d, but are difficult to see within the displayed time interval.) It is much easier to see the updates in Fig. 7b, which compares the OBC and ADS pitch angles. Since the ADS also disregards variations in the integration time interval pitch rates, the estimated pitch angle jumps simultaneously with the onboard solution so that one sees no jumps in their differences. The first impression is that jumps in ground solutions must be smaller, since ground

errors in IRU pitch rates are 8 times smaller than onboard; however, the ground software propagates attitude with erroneous rates during the period 8 times larger than the duration of a minor cycle. Note that the step-wise behavior of errors depicted in Figure 8b is caused by

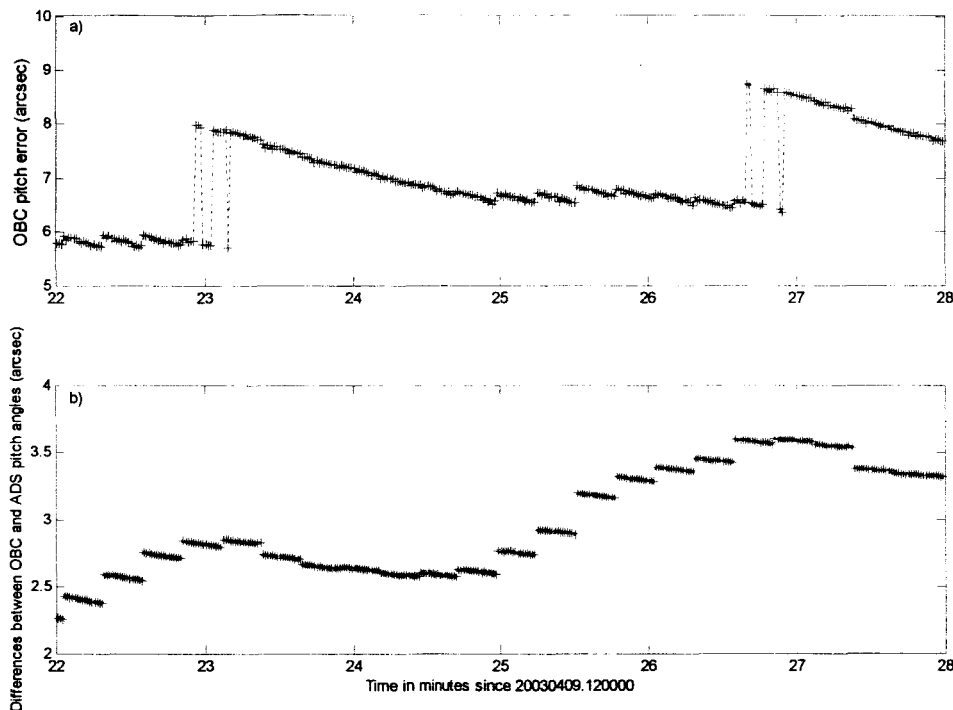


Figure 7. Comparison between the telemetered pitch angle and its ground estimates with and without correction for actual values of the integration time interval.

Onboard inconsistencies between estimated gyro biases and star tracker measurements,⁵ since there are no jumps in IRU measurements.

CONCLUSIONS

It is demonstrated that anomalies observed in Aqua telemetered data can be explained by a small average offset of about $0.50 \pm 0.05 \mu\text{sec}$ in the time intervals between sequential accumulated angle measurements. A scrupulous analysis of the pattern revealed that, despite the fact that the probability curve is noticeably shifted toward positive values, negative time offsets should occur on a regular basis to account for multiple upward and downward jumps in the telemetered pitch rate.

It is also demonstrated that the neglect of variations in the integration time interval results in 2-arcsecond errors in both onboard and ground attitude solutions. A simple formula to adjust ground IRU rates for these variations is suggested and successfully tested.

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